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# COMMENT ON "ON SYMMETRIES AND ANTISYMMETRIES IN SOLVING VIBRATION PROBLEMS USING HIGH PRECISION FINITE ELEMENTS"

In a recent note, Shastry *et al.* [1] considered the use of structural symmetry to reduce the size of vibration (eigenvalue) problems modeled with high precision finite elements (for which the nodal degrees of freedom include not only displacements and rotations, but higher order derivatives as well). The authors described as an "anomaly" the situation in which different answers are obtained depending on whether or not the symmetries and antisymmetries are considered. The purpose of these remarks is merely to point out that no such anomaly exists; rather, if the two analyses yield different results, the conclusion to be drawn is that symmetry and antisymmetry are incorrectly applied. After all, the purpose of exploiting symmetries is to solve (with less effort) the *same* problem rather than a different problem.

Consider the example used in reference [1] of a two-element, simply supported beam for which the lateral displacement variation $w$ within an element is a polynomial of degree seven. Since such a polynomial has eight terms, the element has a total of eight degrees of freedom, four at each end. The four are taken to be $w$, $w'$, $w''$ and $w'''$, where a prime denotes differentiation with respect to $x$.

The authors [1] considered two possible boundary conditions at a simple support,

$$w = 0 \tag{1}$$

and

$$w = w'' = 0. \tag{2}$$

(Physically, the $w''$ condition corresponds to the zero moment condition.) However, regardless of whether expression (1) or (2) is chosen as the end support condition, the symmetry and antisymmetry conditions to be imposed at the center point (when only one-half the structure is modeled) are *independently* determined and have exactly one correct specification.

If the $x$ origin is located at the mid-point of the two-element beam, then the lateral displacement $w(x)$ for symmetric modes is (by definition) an even function: i.e.,

$$w(-x) = w(x). \tag{3}$$

Hence

$$w'(-x) = -w'(x), \tag{4}$$

$$w''(-x) = w''(x), \tag{5}$$

$$w'''(-x) = -w'''(x). \tag{6}$$

In particular, at the point $x = 0$ lying in the plane of symmetry, equations (4) and (6) imply that

$$w' = w''' = 0 \tag{7}$$

for points in the plane of symmetry. By similar arguments for odd functions, it follows that the antisymmetry boundary conditions are

$$w = w'' = 0. \tag{8}$$

That is, equations (7) and (8) are the *required* boundary conditions for symmetric and antisymmetric modes, respectively.

It should be obvious that the set of degrees of freedom constrained as a consequence of symmetry is *complementary* to the set which is constrained due to antisymmetry: i.e., whichever freedoms are constrained for symmetry will be free for antisymmetry, and *vice versa.* This follows directly from the definitions, since the only real distinction between symmetry and antisymmetry is the additional negation that distinguishes even from odd functions.

Thus, if one is able to determine for some problem the (correct) symmetry boundary conditions, the antisymmetry conditions consist of the complementary set of degrees of freedom. One consequence of this property is that the total number of degrees of freedom arising from symmetric and antisymmetric models of a structure must equal the original number of degrees of freedom for the entire structure disregarding symmetry. Thus, it cannot happen (as the authors claim [1]) that the application of symmetry results in the creation of some new degrees of freedom and hence some new eigenvalues. If "conservation of degrees of freedom" is violated by applying too few constraints when symmetry is considered, the lower eigenvalues which result may turn out to be "better" (i.e., closer to exact values) but unfortunately the problem has been redefined in the process.

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REFERENCE

1. B. P. SHASTRY, T. V. G. K. MURTHY and G. VENKATESWARA RAO 1976 *Journal of Sound and Vibration* **47**, 444–446. On symmetries and antisymmetries in solving vibration problems using high precision finite elements.

## AUTHORS' REPLY

The authors thank Dr Everstine for his comments on their note "On symmetries and antisymmetries in solving vibration problems using high precision finite elements". This note was aimed at showing that an anomaly does exist when *only kinematic conditions* are satisfied at the boundaries and lines of symmetry and antisymmetry, according to the displacement formulation, while using high precision elements (i.e., when the elements have not only displacements and their first derivatives but also some higher derivatives as nodal degrees of freedom). The authors are also aware and are in full agreement with Dr Everstine that when *all* symmetric and antisymmetric conditions are satisfied no anomaly exists.

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