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RETARDED POTENTIAL TECHNIQUES FOR THE ANALYSIS OF SUBMERGED STRUCTURES IMPINGED BY WEAK SHOCK WAVES

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ABSTRACT

The retarded potential integral equation is applied in conjunction with the finite element structural analysis techniques to calculate the transient response of submerged structures excited by incident weak shock waves. An example calculation is carried out for the case of a submerged spherical elastic shell impinged by a plane step wave. Comparison of results to an exact solution obtained by the classical separation of variable technique of partial differential equations demonstrates that this approach is highly effective for treating this class of transient fluid-structure interaction problems involving arbitrary but reasonably smooth geometries.

NOMENCLATURE

a = radius of the middle surface of the spherical shell

A^k_{ij} = the influence coefficient for the effect of acceleration at (r_j, t) on the field on S_k at time $(t - i\tau)$

B^k_{ij} = the influence coefficient for the effect of pressure at (r_j, t) on the field on S_k at time $(t - i\tau)$

c = sound speed of the surrounding fluid medium

$\hat{c}$ = unit vector in the direction of propagation of the incident wave

C_q = the qth coefficient of the numerical differentiation formula

E = Young's modulus

I = the maximum value of i for which any B^k_{ij} is nonzero

I' = the maximum value of i for which any A^k_{ij} is nonzero

K = number of zones into which S is divided

L = number of elements into which S_k is subdivided

n_o = unit normal on S into the scattering body

p = time dependent three-dimensional pressure field resulting from the interaction

p^{inc} = incident wave pressure field

p^{sca} = scattered wave pressure field

p^{rad} = radiated wave pressure field

p^{rig} = $p^{inc} + p^{sca}$

$p_t = \frac{\partial p}{\partial t}$

$\mathbf{r}$ = position vector of the observation point

$\mathbf{r}_o$ = position vector of the integration point

$R = |\mathbf{r} - \mathbf{r}_o|$

$R_{jkl} = |\mathbf{r}_j - \mathbf{r}_{kl}|$

$\hat{R}$ = unit vector in the direction from $\mathbf{r}$ to $\mathbf{r}_o$

S = the wetted surface of the submerged structure

S_k = the kth zone of S

S_{jkl} = the ℓth element of S_k as subdivided for the observation from $\mathbf{r}_j$

t = observation time

t_o = retarded time as defined in equation (6)

$\ddot{w}$ = normal acceleration of S in the direction of $\mathbf{n}_o$

(x, y, z) = Cartesian coordinates

γ_{jkl} = the fractional part of time in units of τ as defined in equation (10)

Γ_{jkl} = geometrical coefficients defined by equation (14)

δ_x = shell deflection in the x-direction

$\dot{\delta}_z$ = shell velocity in the z-direction

(θ, ϕ) = spherical coordinates

Λ = number of elements into which ξ is divided

ν = Poisson's ratio

ξ = the discontinuity line defined by the intersection of the bounding surface of the domain of dependence of $\mathbf{r}$ with the scattering surface

ξ_λ = the λth element of ξ

ρ = mass density of the fluid medium

ρ_s = mass density of the shell material

τ = integration time step

$\Psi_{j\lambda}$ = quantity defined by equation (16)

Ω_{jkl} = geometrical coefficients defined by equation (15)

Superscripts

inc = incident

k = kth zone

m = index of time step

rad = radiation

sca = scattered

INTRODUCTION

To calculate the dynamic response of a submerged structure impinged by incident pressure waves, it is necessary to solve simultaneously the structural dynamic, scattering and radiation problems. For this complex fluid-structure interaction problem, the surrounding fluid is often considered to be an ideal compressible medium in linear wave motion. Even under this acoustic treatment, one has not been able to solve the problem analytically for any but the simplest geometries, e.g., for spherical and cylindrical elastic shells. These simple geometries permit the separation of variables in the wave equation and the shell equations of motion leading to standard solutions in the form of normal mode series. These solutions seem to reveal many of the essential phenomena involved in this interaction problem. The physical features and mathematical techniques were recently reviewed in (1).

For cases involving complex submerged structures, numerical methods such as the finite element technique are available as the state-of-the-art methodology for analyzing the structural response. To define the transient interaction loading due to the scattering of the incident wave and the radiation by the structure response motion, various approximate methods have been proposed. They are often based upon the well known asymptotic behaviors of the fluid wave motion, i.e., at early time (high frequency) of the interaction the fluid loading tends to be a damping force and at late time (low frequency) it tends to be an inertial force of added mass. These approximate schemes are recently appraised in (2, 3, 4) and it appears that the Doubly Asymptotic Approximation (DAA) formulated by Geers (5) offers the best compromise between computational cost and accuracy since it correctly calculates the fluid loading at early and late time. The DAA approach has recently been implemented with NASTRAN (6). In the intermediate time region or for the radiation by the structural vibrations in the intermediate frequency range, its accuracy, however, is uncertain (2). The few cases examined (3,4) indicate that this formulation overpredicts the fluid resistance and thus overdamps the structural vibration and causes frequency distortion.

To more accurately calculate the interaction loading, a better treatment of the fluid wave motion is necessary. It is also concluded in (2) that unless nonlinear or real fluid effects need to be taken into account any three-dimensional treatment of the fluid motion is unwarranted because of not only numerical inefficiency but also the fact that the general solution of the wave equation can be represented by the two-dimensional retarded potential integral equation. This equation has been applied to compute transient scattered pressure fields by hard and soft bodies of arbitrary shapes (7-13) and bodies with a Robin boundary condition (14) and to compute the transient pressure fields radiated by bodies of arbitrary shapes (15). The numerical techniques are quite well developed. To include the interaction effects of the structural response of the scatterer, the retarded potential integral equation has to be interfaced with a general method, such as the finite element or finite difference method, for the analysis of the dynamic behavior of a three-dimensional structure. This is attempted here and example results demonstrate that this approach is highly effective.

MATHEMATICAL FORMULATION

Figure 1 schematically sketches a deformable structure of arbitrary shape submerged in an accoustic fluid of infinite extent. The surface S of the structure is piecewise smooth. An arbitrary incident pressure wave p^{inc} impinges on this structure and the interaction can be conceived as the following. The incident wave is scattered by the structure as if it were a hard body and thus creates a scattered pressure wave p^{sca}, while simultaneously the structure's response motion initiates a radiation pressure wave p^{rad} in the surrounding fluid. The total pressure field p is the sum of these three constituent parts, i.e.,

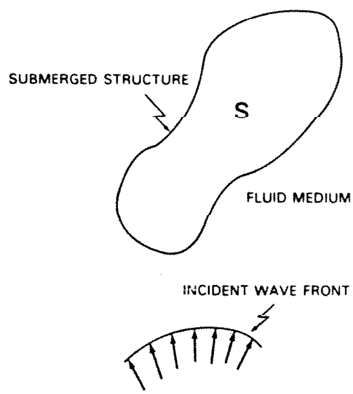

Figure 1. Incidence of a pressure wave on a submerged structure of arbitrary shape

$$p = p^{inc} + p^{sca} + p^{rad} \tag{1}$$

where p and each of its components or any linear combination thereof satisfy the wave equation

$$\nabla^2 p = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}. \tag{2}$$

The boundary condition at the "wetted" surface of the structure can be written as (1)

$$\frac{\partial p}{\partial n_o} = -\rho \ddot{w}. \tag{3}$$

From physical considerations, p^{sca} and p^{rad} satisfy a radiation condition sufficiently far away from the structure. They would be at least piecewise continuous and differentiable functions of space and time so that their solutions can be represented by the retarded potential integral equation. For the case where the scatterer or the radiator is surrounded by a fluid medium of infinite extent, the uniqueness of the retarded potential representation has recently been demonstrated analytically by Gerdes and Martensen (16).

For the present application, it can be readily shown that, by using equations (1), (2) and (3), the scattered and radiation fields at the structure surface S can be represented separately as the following.

$$p^{rad}(r,t) = -\frac{\rho}{2\pi}\iint_S \frac{\ddot{w}(r_o,t_o)}{R}\,dS + \frac{1}{2\pi}\iint_S \left[p^{rad}(r_o,t_o) + \frac{R}{c}\frac{\partial}{\partial t_o}p^{rad}(r_o,t_o)\right]\frac{1}{R^2}\frac{\partial R}{\partial n_o}\,dS \tag{4}$$

$$p^{rig}(r,t) = 2p^{inc}(r,t) + \frac{1}{2\pi}\iint_S \left[p^{rig}(r_o,t_o) + \frac{R}{c}\frac{\partial}{\partial t_o}p^{rig}(r_o,t_o)\right]\frac{1}{R^2}\frac{\partial R}{\partial n_o}\,dS + \frac{1}{2\pi}\int_\xi \frac{p_f}{|(\hat{R}+\hat{c})\times n_o|}\frac{1}{R}\frac{\partial R}{\partial n_o}\,d\xi \tag{5}$$

where the retarded time

$$t_o = t - \frac{R}{c} \tag{6}$$

and

$$p^{rig} = p^{inc} + p^{sca}. \tag{7}$$

Equation (5) is for the calculation of the pressure field due to the scattering by a hard body and the line integral term on the right hand side is necessary whenever the incident wave fronts are discontinuous in pressure and velocity (acoustic shock waves) (7,14). This line integral in the present form is derived by Neilsen et al. (13) in terms of local time rate of outward movement of the discontinuity on the scattering surface and the local orientation of the scattering surface at r, thus enabling development of numerical integration schemes to accommodate a wide variety of scattering surface shapes. The line integral is to be taken around the discontinuity line ξ which is defined by the intersection of the bounding surface of the domain of dependence of point r with the scattering surface. The domain of dependence of point r is the region of space containing all points from which scattered waves due to the passage of the incident wave can reach point r at time t.

In equation (5), p_f denotes the pressure at the wave front on the surface. It is due to incidence and local reflection of the wave front only and can be determined by geometric acoustic theory, i.e., it is twice the incident wave front pressure on the illuminated surface, equal to the wave front pressure on a surface element parallel to the direction of incidence, and zero in the shadow region. The solution of equation (5) involves only the geometry of the scatterer and it is equation (4) involves the structural response of the scatterer through $\ddot{w}$ term on its right hand side and need to be interfaced with the structural dynamic analysis. The separate treatment of p^{rig} and p^{rad} is both advantageous and necessary since equation (5) requires different numerical treatment if the incident wave has a discontinuous pressure front (7, 13, 14). In applications, p^{rig} is computed separately for the given properties of the incident wave and of the geometry of the submerged structure. The result is stored as a part of the time dependent pressure loading.

For numerical computations, the integrals in equations (4) and (5) are discretized. A general treatment of the surface integral has been formulated by Mitzner (11) who divides S into K zones S_k and assumes that the pressure p_k its time derivative $p_{t,k}$ and therefore $\ddot{w}_k$ are uniform everywhere on S_k. The S_k's are then subdivided into L elements S_{jkl} which are sufficiently small. For the line integral, the discontinuity line ξ for each observation time t is divided into Λ segments ξ_λ. A sufficiently small time interval τ is chosen such that

$$\min \{R_{jkl}\}_{j \neq k} \geqslant c\tau \tag{8}$$

$$t = m\tau \tag{9}$$

$$\hat{t}_{jkl} = (n_{jkl} + \gamma_{jkl})\tau, \quad n \text{ integer}, \quad 0 \leqslant \gamma < 1 \tag{10}$$

p_k and $p_{t,k}$ at $\hat{t}_{jkl}$ can be interpolated, if $\hat{t}_{jkl}$ is not a multiple of τ, by the two-point approximations formula of the form

$$p_k(m\tau - \hat{t}) = (1-\gamma)p_k^{(m-n)} + \gamma p_k^{(m-n-1)}. \tag{11}$$

The time derivative of p_k is expressed by the standard three-point backward difference formula.

$$(p_{t,k})^m = \frac{1}{\tau}\sum_{q=0}^{2} C_q p_k^{m-q} \tag{12}$$

where

$$C_o = \frac{3}{2}, \quad C_1 = -2, \quad C_2 = \frac{1}{2}. \tag{13}$$

The pressure at the jth observation point at time t can now be expressed by the following discretized forms

$$\begin{aligned}\left(p_j^{rad}\right)^m = & -\frac{\rho}{2\pi}\sum_{k=1}^{K}\sum_{\ell=1}^{L} \Gamma_{jkl}\left[(1-\gamma_{jkl})\ddot{w}_k^{(m-n)} + \gamma_{jkl}\ddot{w}_k^{(m-n-1)}\right] \\ & -\frac{1}{2\pi}\sum_{k=1}^{K}\sum_{\ell=1}^{L} \Omega_{jkl}\left\{(1-\gamma_{jkl})\left[\left(p_k^{rad}\right)^{(m-n)} + \frac{\hat{t}_{jkl}}{\tau}\sum_{q=0}^{2} C_q\left(p_k^{rad}\right)^{(m-n-q)}\right]\right. \\ & \left. +\gamma_{jkl}\left[\left(p_k^{rad}\right)^{m-n-1} + \frac{\hat{t}_{jkl}}{\tau}\sum_{q=0}^{2} C_q\left(p_k^{rad}\right)^{(m-n-q-1)}\right]\right\}\end{aligned} \tag{4a}$$

$$\begin{aligned}\left(p_j^{rig}\right)^m = & \, 2\left(p_j^{inc}\right)^m - \frac{1}{2\pi}\sum_{k=1}^{K}\sum_{\ell=1}^{L} \Omega_{jkl}\left\{(1-\gamma_{jkl})\left[\left(p_k^{rig}\right)^{(m-n)} + \frac{\hat{t}_{jkl}}{\tau}\sum_{q=0}^{2} C_q\left(p_k^{rig}\right)^{(m-n-q)}\right]\right. \\ & \left. +\gamma_{jkl}\left[\left(p_k^{rig}\right)^{(m-n-1)} + \frac{\hat{t}_{jkl}}{\tau}\sum_{q=0}^{2} C_q\left(p_k^{rig}\right)^{(m-n-q-1)}\right]\right\} \\ & -\frac{1}{2\pi}\sum_{\lambda=1}^{\Lambda} \Psi_{j\lambda}\end{aligned} \tag{5a}$$

where

$$\Gamma_{jkl} = \iint_{S_{jkl}} \frac{dS}{R_{jkl}} \tag{14}$$

$$\Omega_{jkl} = -\iint_{S_{jkl}} \frac{1}{R_{jkl}^2} \frac{\partial R_{jkl}}{\partial n_o} dS \tag{15}$$

$$\Psi_{j\lambda} = -\int_{\xi_\lambda} \frac{(p_f)_\lambda}{|(\hat{R}_{j\lambda} + \hat{c}) \times n_o|} \cdot \frac{1}{R_{j\lambda}} \frac{\partial R_{j\lambda}}{\partial n_o} d\xi . \tag{16}$$

These are to be evaluated numerically. A general procedure for Ω_{jkl} has been devised by Mitzner (11,12). Hess and Smith (17) also comprehensively report their calculation methods for Γ_{jkl} and Ω_{jkl}. A numerical scheme for tracing the discontinuity line on the scatter and for the computation of $\Psi_{j\lambda}$ is reported in detail in reference (13). The summation over ℓ can be easily rearranged for machine computation as a summation over the time delay index i (11,12), so that equation (4a) assumes the following form

$$\left(p_j^{rad}\right)^m = -\frac{\rho}{2\pi} \sum_{k=1}^{K} \sum_{i=0}^{I'} A_{ij}^k \ddot{w}_k^{(m-i)} - \frac{1}{2\pi} \sum_{k=1}^{K} \sum_{i=0}^{I} B_{ij}^k \left(p_k^{rad}\right)^{m-i} \tag{4b}$$

where

$$I' = \max(R_{jkl})/(c\tau) + 1 \tag{17}$$

$$I = \max(R_{jkl})/(c\tau) + 3 . \tag{18}$$

For the arrangement of equation (4b), A_{ij}^k and B_{ij}^k depend only on the geometry and thus can be computed and stored prior to calculating p^{rad}. The numbers of coefficients of A_{ij}^k and B_{ij}^k needed to be stored are less than half of $(I'K^2 + K)$ and $(IK^2 + K)$ respectively. Equations (4a), (4b) and (5a) are explicit and their solutions are to be carried out step by step timewise. Therefore, in the calculation of the transient response of the submerged structure, equation (4b) (or (4a)) can be conveniently interfaced with a structural analysis method which uses a direct integration technique to obtain time dependent solutions. The finite difference form of the equation of motion of the structure can be solved simultaneously with equation (4b). Due to the retarded time effect, $(p_j^{rad})^m$ depends on only a few of the $\ddot{w}_k^m$'s. For applications with existing finite element structural analysis computer programs, however, the following iterative scheme seems convenient if the integration time step τ is sufficiently small. The p_k^{rig}'s for all time steps are separately computed and form part of the input loading in the structural program. Starting from the initial time step (i = 0), the $w_k^{(i)}$'s are computed and transferred to a subprogram for equation (4b). The $(p_k^{rad})^{(i+1)}$'s are then computed, transferred to the structural program and added to the $(p_k^{rig})^{(i+1)}$'s to form the total loading $(p_k)^{(i+1)}$ for computing the $\ddot{w}_k^{(i+1)}$'s. This process is repeated until all time steps have been advanced.

NUMERICAL RESULTS AND DISCUSSION

To demonstrate the validity, accuracy and practicality of this approach, the described method is applied to calculate the transient response of a steel spherical elastic shell submerged in water and impinged by an incident plane step wave. Figure 2 schematically sketches the geometry of this problem. The transient scattering pressure of the plane step wave by a hard sphere has been computed based on equation (5a) by Huang in reference (13). When the results are compared to the closed form series solution obtained by the classical separation of variables and Laplace transform method in reference (18), it is observed that the retarded potential solution in general closely agrees with the series solution and improves over it by accurately describing the discontinuous scattered wave front on the sphere at early-time. For convenience, the closed form series solution for p^{rig} is used for the present test problem.

Equation (4b) is interfaced, using the described iterative scheme, with the general purpose finite element structural analysis computer program, NASTRAN (19). Since the problem is axisymmetric with respect to the z-axis, the elastic sphere is represented by 20 NASTRAN

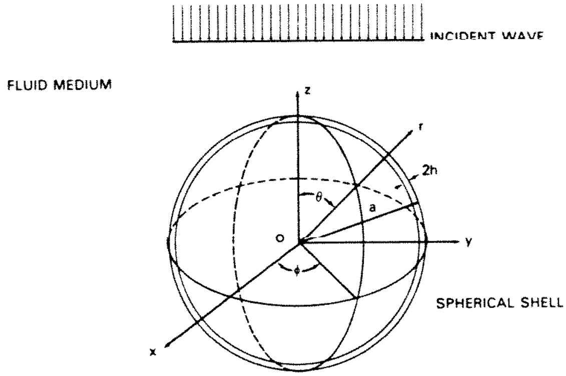

Figure 2. Transient interaction of a spherical elastic shell with a plane step incident pressure wave

axisymmetric conical shell elements whose boundaries are the $\theta = 9k^\circ$ ($k = 0, 1, 2, \ldots 20$) lines. The elements have different areas and it is also necessary to leave two small holes at $\theta = 0$ and π for the two pole elements. The surface of each element also constitutes a zone for the retarded potential integral in equation (4a). For p^{rad}, the zones are further subdivided using fine grids and coarse grids. For the singular zones ($j = k$), pole zones and the cases $R_{jkl} < 1.5a$, the kth zone is subdivided by five equal divisions in θ and 75 equal divisions in ϕ. Otherwise, a zone is subdivided into 15 elements by 15 equal ϕ-divisions. These and the treatment of the singular zones closely follow the procedure in reference (12). The time step τ used corresponds to $ct/a = 0.05$. The "cylindrical" storage routine of references (11,12) is also used for A^k_{ij} and $B^k_{\nu j}$ here. In NASTRAN, the equations of motion of the 21 grid circles are integrated by a variant of the Newmark-beta method. The diameter to thickness ratio of the spherical shell is 100. The properties of the steel are: Young's modulas $E = 2.0684 \times 10^{11}$ Pa, Poisson's ratio $\nu = 0.3$ and mass density $\rho_s = 7784.5$ kg/m^3. The properties of the surrounding water are sound speed $c = 1461.2$ m/s and mass density $\rho = 999.6$ kg/m^3.

Results are compared to an exact solution obtained by the classical separation of variables method in (18). The time histories of deflections in the x-direction and velocities in the z-direction at various locations of the elastic shell are exhibited in Figures 3, 4 and 5. Except at $\theta = \pi$, the results obtained by NASTRAN-Retarded-Potential closely match those computed by the classical method. The shell velocity at $\theta = \pi$ presently calculated has the correct oscillation characteristics but the peaks are somewhat too high compared to the classical solution in Figure 5. In general, the accuracy of the present NASTRAN-Retarded-Potential results is highly satisfactory for a time span equal to about 8 transit times for the incident wave front. Thereafter, some "numerical noise" begins to appear.

For the manner of dividing and subdividing zones and the size of integration time step used in the present test calculation, it is necessary to store 6834 coefficients for A^k_{ij} and 7634 for B^k_{ij}. The computations required in equation (4b) are simple multiplications and summations and are quite fast. For problems involving large numbers of structural elements, the size of A^k_{ij} and B^k_{ij} could be very large so that to efficiently apply the present approach, it is necessary to adroitly manipulate the computer peripheral storage to handle these geometrical coefficients. With the present computer technology as well as the trend of its advancement, no technical difficulty is foreseen. The use of the retarded potential technique and the finite element method appears to be offering an excellent opportunity for the analysis of complex submerged structures.

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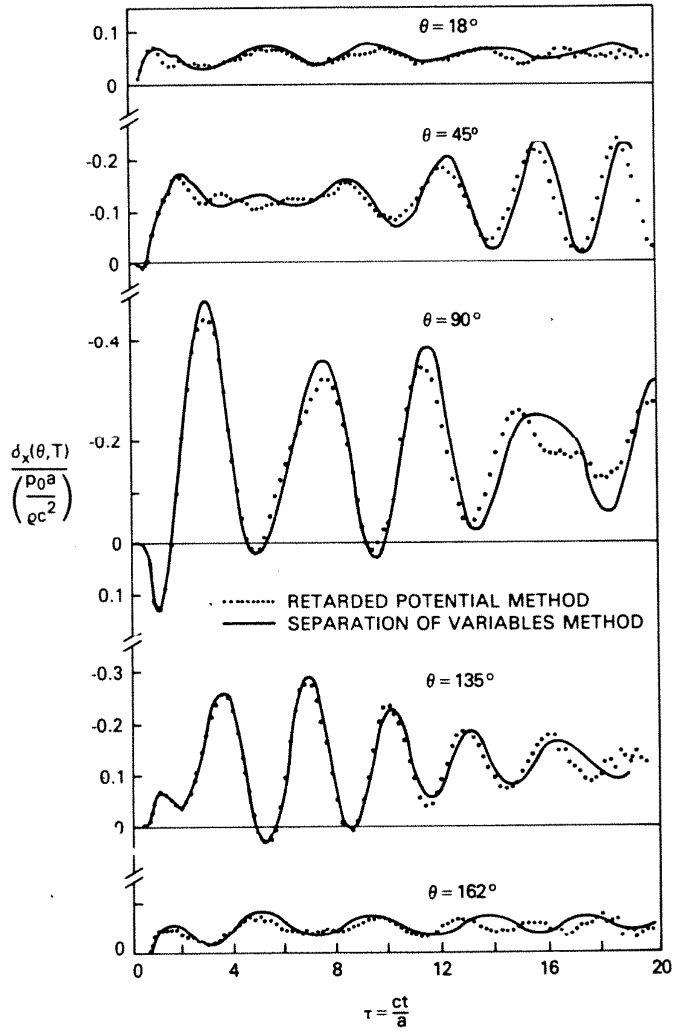

Figure 3. Time histories of shell deflection in the x-direction, ——— classical solution, ... present results

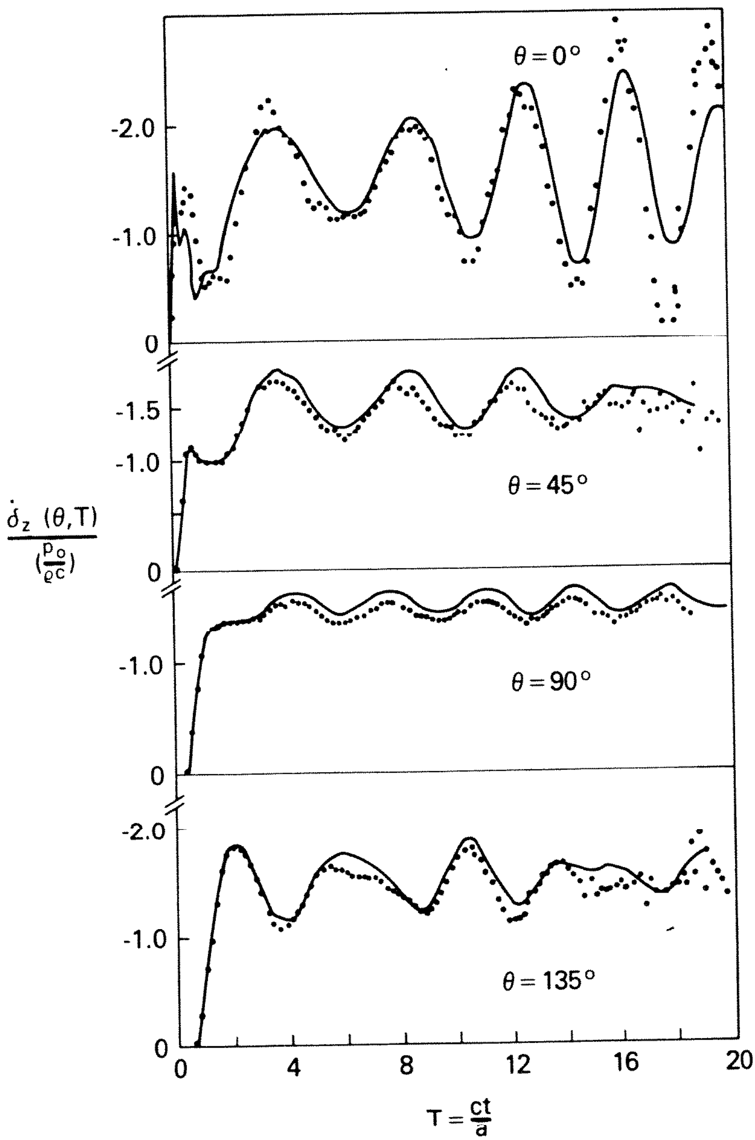

Figure 4. Time histories of shell velocity in the z-direction, ______ classical solution, ... present results

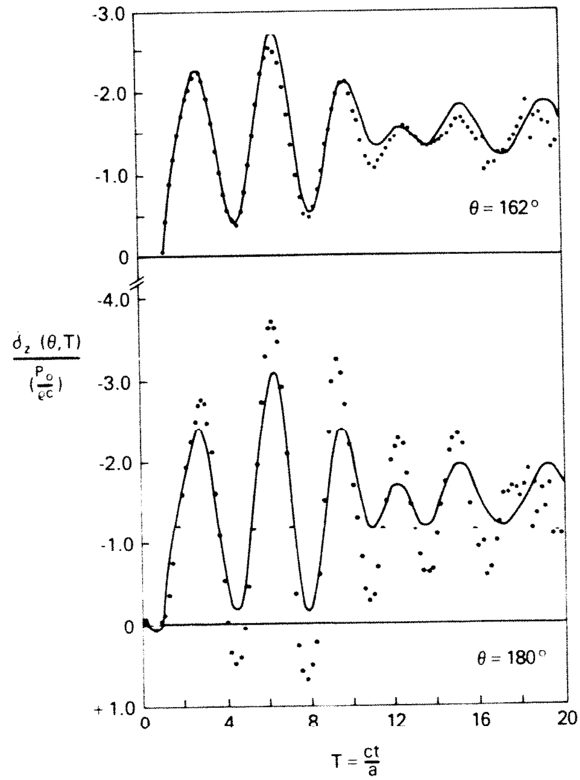

Figure 5. Time histories of shell velocity in the z-direction, ——— classical solution, ... present results

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