Journal of Sound and Vibration (1979) **64**(1), 63–71

VIBRATIONS OF THREE LAYERED DAMPED SANDWICH PLATE COMPOSITES†

Y. P. LU, J. W. KILLIAN AND G. C. EVERSTINE

David W. Taylor Naval Ship Research and Development Center, Bethesda, Maryland 20084, *U.S.A.*

(*Received* 30 *June* 1978, *and in revised form* 23 *December* 1978)

Theoretical and experimental results are presented and discussed for the transverse driving point mechanical impedances, as well as for the transfer impedances, of damped composite plates made up of a thin viscoelastic layer sandwiched between two elastic layers. Analytical results are determined by finite element approximations. Due to the elements used and the system to be modeled, several fundamental assumptions or restrictions usually adopted in analytical investigations are removed. The dependence on frequency and temperature of the dynamic properties of the viscoelastic materials is taken into consideration. A companion experiment was conducted, for comparison purposes, on such damped composite plates suspended in air by lightweight elastic shock cords and driven at the center by an electromechanical vibration shaker. Good correlations between the test data and analytical solutions are obtained over a wide frequency range for two configurations.

1. INTRODUCTION

It has long been observed that structural noise and vibration can be reduced by utilizing layers of viscoelastic shear damping material to dissipate energy within vibrating members. One approach is the constrained layer damping treatment, in which a thin layer of viscoelastic damping material is sandwiched between the primary vibrating structure and a constraining layer. A combination of viscoelastic damping material and metal will provide both strength and rigidity but with a low response to vibration. The soft viscoelastic damping layer undergoes a periodic shear deformation which dissipates energy.

Due to their importance in practical applications, the vibrations of three-layered damped plate composites have been for years the subject of many investigations to determine the loss factors and responses of the systems. An analysis of the effect of this shear damping mechanism on plate vibrations was first given by Kerwin [1] in an essentially one-dimensional formulation in which the boundary conditions were not taken into account. Later, this analysis of plate damping was verified experimentally by Yin, Kelly and Barry [2]. The success of the verification was due to the fact that all plate composite samples used were long strips for which the one-dimensional assumptions were satisfied. In recent years, analyses of three-layered damping sandwich plates have been made by many investigators [3–6], most of whom have assumed that the plate composite structures vibrate in sinusoidal modes of transverse flexural displacement which cause the core to be sheared and energy to be dissipated. Such sinusoidal modes can be achieved only if the plate composites are rectangular and have simply supported edges. Perhaps this is why little work has been done on the analysis of damped plate structures with end conditions other than simply supported. Moreover, most previous work on such three-layered plates has, unfortunately,

† All opinions or assertions made in this paper are those of the authors and are not to be construed as official or necessarily the views of the Navy or the naval service at large.

0022-460X/79/090063 09 $02.00/0

not included experimental data. Since the range of validity for many of the assumptions used in the theories is not well established, such test data would be valuable.

Analyses developed for simple damped geometries, such as sandwich plates, indicate that the complexity of analyses for boundary conditions other than simply supported or for more complicated geometries will be drastically increased, and that the solutions for practical systems of this nature would be very difficult to obtain. For this reason, researchers have developed various finite element approaches to the analysis of layered damped structures. Among others, Mau, Pian and Tong [7] have demonstrated the feasibility and accuracy for laminated plates of finite element solutions with hybrid stress elements. Chan and Cheung [8] have solved the multi-layered sandwiched plate problems by the finite strip method.

A recent paper by Lalanne, Paulard and Trompette [9] presented a method based on a finite element technique for the prediction of the harmonic response of thick damped structures. Good comparisons between analytical and experimental results for layered plates and layered beams were given. The solutions were determined in terms of modal co-ordinates instead of physical co-ordinates. In their excellent and simple approach it is assumed that every frequency in the undamped system has a corresponding frequency in the damped system. In practice, however, the effect of damping is to suppress resonant peaks and clustering is observed. Resonant frequencies still exist though they may not be associated with a resonant peak. Thus, they may not be readily seen. They all contribute to the very difficult problem of identifying the resonant frequencies and associated modes. In the present paper, the generalized co-ordinates approach is used with a general purpose computer program requiring no experimental information other than the geometrical configurations and material properties.

Since one of the primary objectives of employing a damped composite structure is to attenuate the vibratory response under resonant conditions, it is important to find the resonant frequencies and the associated mode shapes of such a system. However, since the viscoelastic material properties are frequency-dependent, it is a formidable task to obtain such information. This is one of the most significant difficulties in dealing with the sandwich damped systems.

This paper is concerned with damped three-layered sandwich plates subjected to a time-harmonic transversely concentrated load. The sandwich plate is made up of two elastic layers with a thin viscoelastic damping core between them (Figure 1). Both materials are considered isotropic and homogeneous. The primary objective of our work is to develop an experimentally verified analytical model, via a finite element approach, for such a

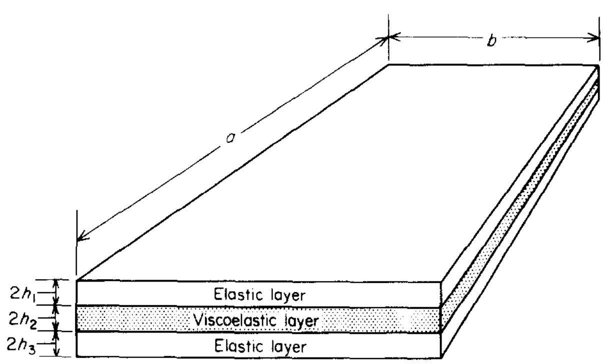

Figure 1. Three-layered damped laminated plate.

damped system and to use the model in design applications to study how the vibration attenuation varies with frequency and geometry. Typical experimental and theoretical results of the mechanical driving point impedances, and transfer impedances of two composite plate samples, are presented and discussed. The analytical solutions were obtained by using the NASTRAN finite element structural analysis computer program [10, 11]. Due to the elements used and the system modeled, several fundamental assumptions or restrictions usually adopted in analytical investigations are removed. Theoretical predictions compare very well with experimental data over a wide frequency range.

2. EXPERIMENTAL PROCEDURES

Two damped composite plates were constructed to evaluate experimentally the numerical solutions obtained by using the NASTRAN finite element computer program. Specimen 1 incorporated a 0·006 inch ($1{\cdot}5 \times 10^{-4}$ m) acrylic base viscoelastic polymer material, with mass density $1{\cdot}03 \times 10^{-4}$ lb s^2/in^4 ($2{\cdot}851$ kg/m^3), sandwiched between a machined steel primary vibrating structure, $\frac{1}{2}$ inch ($1{\cdot}27 \times 10^{-2}$ m) thick, and a steel constraining layer $\frac{1}{8}$ inch ($0{\cdot}3175 \times 10^{-2}$ m) thick. The machined plate composite was 14 inches × 14 inches (0·3556 m × 0·3556 m) in lateral dimensions. Specimen 2 was similar except that the primary structure was 1 inch ($2{\cdot}54 \times 10^{-2}$ m) thick, and the constraining layer was $\frac{1}{4}$ inch ($0{\cdot}635 \times 10^{-2}$ m) thick. The machined plates help ensure a uniform cross-section for the thin damping core. Both specimens were tested at 68°F (20°C). The dynamic material properties of the viscoelastic layer over the frequency range of 50 Hz to 1000 Hz were supplied by the manufacturer.

The complex shear modulus G at this temperature can be approximated by

$$G = G'(1 + i\beta) \tag{1}$$

where

$$G' = (102{\cdot}39 \times 10^4)f^{0{\cdot}625} \text{ (Pascals)}, \qquad \beta = 1{\cdot}6274 f^{-0{\cdot}072}. \tag{2, 3}$$

In these equations, f is the frequency in Hertz. Examination of the temperature and frequency dependence of the complex shear modulus shows that this material approximates a class A thermorheologically simple material. On the basis of this assumption, the complex shear modulus was extended to 5000 Hz.

The test conducted was similar to that described previously [12, 13]. The specimen was suitably suspended in air by using a lightweight elastic shock cord. This arrangement resulted in a system natural frequency well below 5 Hz and closely approximated free boundary conditions for the frequencies of particular interest. There were two reasons for such a set-up: first, the freely suspended condition was convenient for conducting experiments, and second, this boundary condition differed from the simply supported condition used by most authors. The square composite plates were excited at the center of the plate by an electromechanical vibration shaker. Between the shaker and the plate composite, a mechanical impedance head was placed that yielded the driving point impedance, which is defined as the ratio of the applied force to the velocity produced at the point of excitation†. The force and acceleration signals from the impedance head were analyzed by 2 Hz bandwidth tracking filters and combined logarithmically to produce a signal proportional to

† Although this is the customary definition, the actual quantity measured experimentally or computed analytically is the inverse of the driving point mobility.

impedance. The frequency range was swept slowly by a sweep oscillator so that a continuous spectrum was obtained. The sweep rate was slow enough to allow both the structure and the electronics to respond properly and to record the changes in mechanical impedance. The impedance signal was fed to an automatic impedance plotter, yielding a logarithmic plot of the impedance versus the frequency of excitation. The transfer impedance plots were similarly obtained at specific locations for the same driving force applied at the center of the composite.

3. FINITE ELEMENT APPROXIMATIONS AND NUMERICAL RESULTS

The numerical results were obtained by means of NASTRAN's direct frequency response analysis capability. In the system investigated, the viscoelastic material is assumed to be nearly incompressible. Since this assumption is approximately true for most viscoelastic materials [14], there is no difference between the values for the loss factors in shear and those in direct deformation. Poisson's ratio is taken as 0·48 to avoid numerical difficulties. The complex shear modulus of elasticity is both temperature- and frequency-dependent according to equation (1). The elastic layers are assumed to be lossless.

In obtaining the NASTRAN solutions for the damped systems under consideration, the plate is excited at the center of the composite, and the plate boundaries are assumed to be completely free. Only a quarter of the composite is modeled because of symmetry, with symmetry boundary conditions applied as displacement constraints. The viscoelastic damping core is modeled with linear isoparametric brick elements having eight nodes. The primary base plate and the constraining layer are modeled with homogeneous quadrilateral plate elements having four nodes. Each layer is modeled with a 5×5 array of elements. Thus, there are four layers of grid points, one each on the middle plane of the elastic plates, and one each on the upper and lower surfaces of the viscoelastic layer. Each plate is then rigidly connected to the viscoelastic layer. The primary vibrating member of the composite is assumed to be the bottom plate.

The model described above includes the effects of transverse shear flexibility, translatory and transverse inertias (both in the face plates and the core), and rotatory inertia and

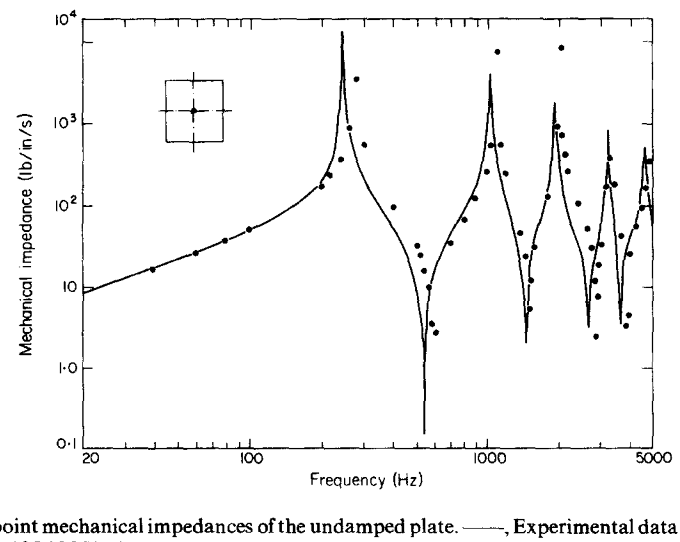

Figure 2. Driving point mechanical impedances of the undamped plate. ——, Experimental data; · · · , NASTRAN solutions. 1 lb/in/s = 125·13 N/m/s.

extensional effects in the viscoelastic core. It is not assumed. however, that at any point in the plates both the elastic and the viscoelastic layers displace normal to the plate by the same amount. Other common assumptions, such as equal facing thicknesses. small damping, and small bending stress in the core are also removed. Some of these effects have often been adopted as basic assumptions or restrictions in the analytical investigations of systems such as those under consideration.

In order to evaluate the accuracy of the test results, the responses were first determined for the undamped plate (i.e., the primary vibrating member of the composite system of specimen 1), since obtaining NASTRAN solutions for such a system is straightforward. Figure 2 shows the correlation between experimental data and NASTRAN solutions for the simple undamped system. This figure is a plot of absolute values of mechanical driving point impedance versus frequency over the frequency range of interest. The good correlation establishes the accuracy and the validity of the experimental data measured. Figure 3 gives, for the same system, the frequencies and nodal patterns of the first four excited modes, which are necessarily symmetric with respect to both the coordinate axes and the diagonals. The frequencies and the associated nodal patterns were obtained by using NASTRAN's normal modes analysis.

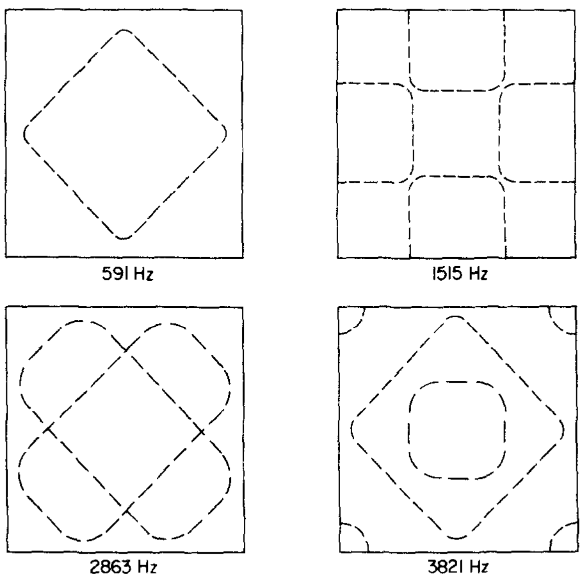

Figure 3. Mode shapes of a completely free square undamped plate.

For the damped plate, Figure 4 shows the excellent correlation between NASTRAN solutions and experimental results for the driving point mechanical impedances of specimen 1. The low frequency response associated with the rigid body motion lies along the mass line of the impedance plot. The impedances presented cover the first four resonant frequencies. Although the calculated frequencies are shifted slightly to the right of the test results, they are all consistent. One possible explanation of the discrepancy between computed and experimental results above 1000 Hz could be that the properties of the viscoelastic material are known only up to 1000 Hz and are extrapolated beyond 1000 Hz. In addition to the excellent correlation, this figure also shows the good damping characteristics of the system due to the presence of the viscoelastic damping core and the constraining plate.

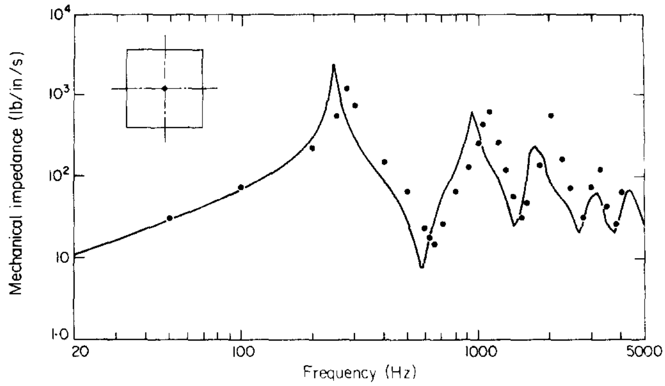

Figure 4. Driving point mechanical impedances of the damped plate (Specimen 1). Legend as for Figure 2.

In Figures 5(a) and (b), transfer impedances for specimen 1 are shown for the two locations indicated on the figures. Both measurement points are located on the diagonal: one (point A) at a distance of 3·96 inches (0·1006 m) from the plate center (Figure 5(a)), and the other (point B) at 5·94 inches (0·1509 m) from the center (Figure 5(b)). In these figures,

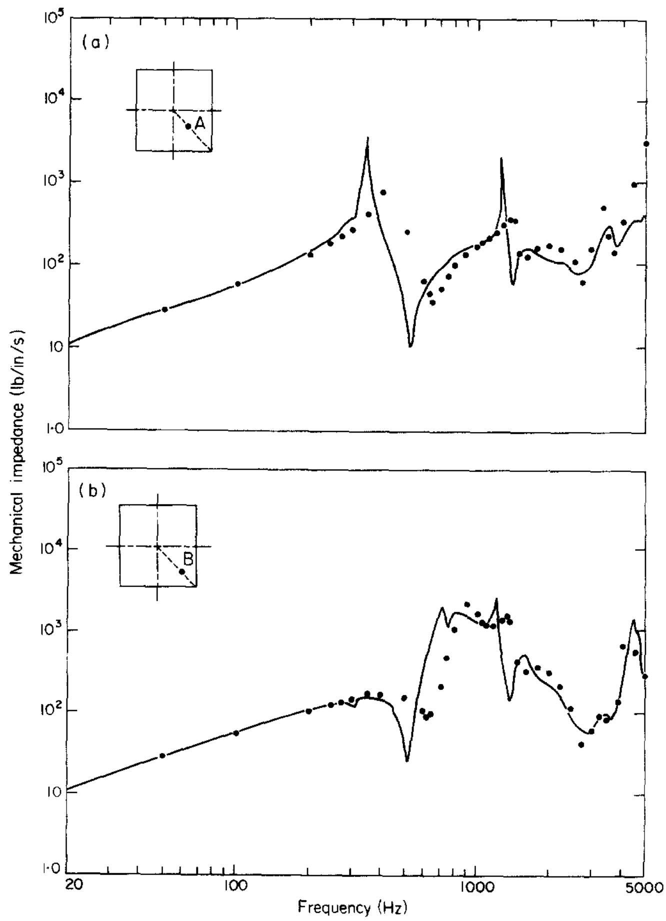

Figure 5. Transfer impedances of the damped plate (Specimen 1): (a) point A; (b) point B. Legend as for Figure 2.

the transfer impedances are generally higher than the driving point impedances. The figures also suggest that the transfer impedances increase with distance from the driving point. This clearly demonstrates the response attenuation produced with the damped system.

Figure 6 presents the mechanical driving point impedances for specimen 2 at a temperature of 68°F (20°C). The irregularity which occurs at approximately 2000 Hz is an error introduced by the particular instrumentation used. Figures 7(a) and (b) show the transfer impedances for the same system at locations identical to those given in Figures 5(a) and (b), respectively. Since specimen 2 has vibratory characteristics similar to those of specimen 1, all interpretations given for previous figures apply here also.

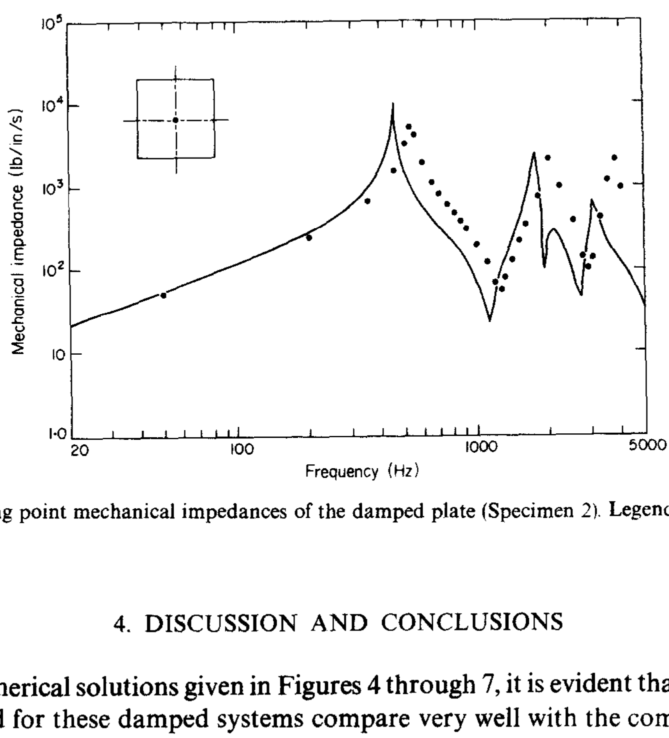

Figure 6. Driving point mechanical impedances of the damped plate (Specimen 2). Legend as for Figure 2.

4. DISCUSSION AND CONCLUSIONS

From the numerical solutions given in Figures 4 through 7, it is evident that the analytical results obtained for these damped systems compare very well with the companion experimental data. It is recognized that the numerical solutions can be further improved by using a finer mesh. Although we are dealing here with a simple geometry, the significance of these results is that they demonstrate that a finite element program such as NASTRAN can be used "as is" to solve more complex and practical problems of damped structures. It is certainly an advantage to be able to use a widely available, well maintained computer program. For such programs, for example, many preprocessors and post-processors are often available [15], including matrix bandwidth and wavefront reduction [16]. The authors are not aware of any previous similar use of finite element programs for problems such as these.

The major difference between these damped systems and other elastic undamped layered structures analyzed by the finite element method is that the results of these damped problems can be obtained for only one frequency at a time because the shear modulus and the loss factors of the viscoelastic materials are functions of frequency for a given temperature. As a result, NASTRAN's standard normal mode analysis can not be used to determine the resonant frequencies and mode shapes of such systems. Hence the forced responses of

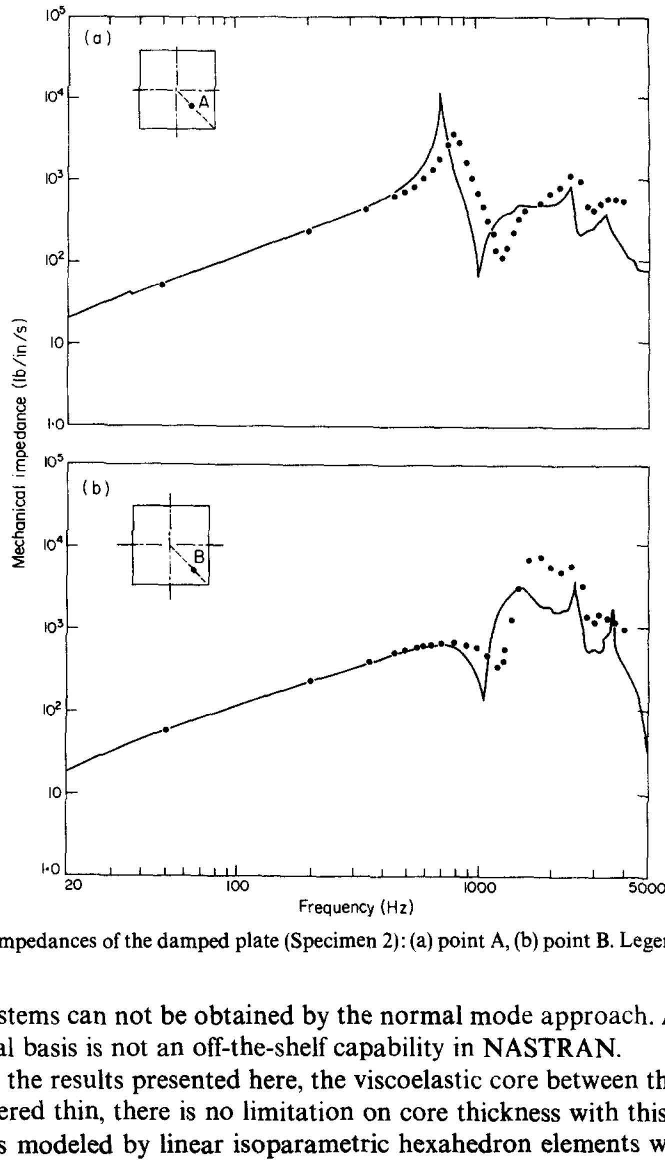

Figure 7. Transfer impedances of the damped plate (Specimen 2): (a) point A, (b) point B. Legend as for Figure 2.

such damped systems can not be obtained by the normal mode approach. Also, the use of a complex modal basis is not an off-the-shelf capability in NASTRAN.

Although, for the results presented here, the viscoelastic core between the elastic plates has been considered thin, there is no limitation on core thickness with this approach, because the core is modeled by linear isoparametric hexahedron elements which are three-dimensional in nature and can also, of course, be used in thick layer applications. In modeling the system, no restrictions with regard to the rigidity and thicknesses of the core and faces are imposed, so that the model is valid for all elastic laminated structures or elastic-viscoelastic-elastic composites with three layers. Clearly, this model can be simplified for application to one-dimensional beam composites. It can also be applied to similar curved plate or beam damped structures. In addition, the layers may be anisotropic.

For generality and for establishing the accuracy and adequacy of the method presented, the value of the stiffness matrix for each frequency has been changed because the material properties are functions of frequency for a given temperature. With the excellent correlation demonstrated in this paper, means should be sought to reduce the computing time by making appropriate assumptions, such as constant Poisson's ratio for all frequencies of interest, or constant shear modulus for a given frequency range. With such assumptions, the computing time for calculating stiffness matrices can be greatly reduced. The approach given here is a necessary first step toward establishing the feasibility of the method.

To summarize, finite element solutions and companion experimental data have been obtained for three-layered damped plate composites made up of two elastic layers with a thin viscoelastic damping core between them. The correlation between the analytical and test results is excellent over a wide frequency range. The finite element model presented in this paper can be relied upon to yield reasonably accurate predictions of such constrained-layer damping sandwich plates.

REFERENCES

1. E. M. Kerwin, Jr. 1959 *Journal of the Acoustical Society of America* **31**, 952–962. Damping of flexural waves by a constrained viscoelastic layer.
2. T. P. Yin, T. J. Kelly and J. E. Barry 1967 *Journal of Engineering for Industry* **89**, 773–784. A quantitative evaluation of constrained-layer damping.
3. R. A. DiTaranto and J. R. McGraw, Jr. 1969 *Journal of Engineering for Industry* **91**, 1081–1090. Vibratory bending of damped laminated plates.
4. D. J. Mead 1971 *STROJNICKY CASOPIS* **XXII**, 53–67. The vibration characteristics of damped sandwich plates with stiffeners and various boundary conditions.
5. M. J. Yan and E. H. Dowell 1972 *Journal of Applied Mechanics* **39**, 1041–1046. Governing equations for vibrating constrained-layer damping sandwich plates and beams.
6. Y. V. K. Sadasiva Rao and B. C. Nakra 1974 *Journal of Sound and Vibration* **34**, 309–326. Vibrations of unsymmetrical sandwich beams and plates with viscoelastic cores.
7. S. T. Mau, T. H. H. Pian and P. Tong 1973 *American Institute of Aeronautics and Astronautics Journal* **11**, 1450–1452. Vibration analysis of laminated plates and shells by a hybrid stress element.
8. H. C. Chan and Y. K. Cheung 1972 *International Journal of Mechanical Science* **14**, 399–406. Static and dynamic analysis of multi-layered sandwich plates.
9. M. Lalanne, M. Paulard and P. Trompette 1975 *Sound and Vibration Bulletin* **45**, 65–72. Response of thick structures dampled by viscoelastic material with application to layered beams and plates.
10. *Anon* 1976 *NASA SP*-221 (03). The NASTRAN theoretical manual.
11. *Anon* 1976 *NASA SP*-222 (03). The NASTRAN user's manual.
12. Y. P. Lu, B. E. Douglas and E. V. Thomas 1973 *American Institute of Aeronautics and Astronautics Journal* **11**, 300–304. Mechanical impedance of damped three-layered sandwich rings.
13. R. A. DiTaranto, Y. P. Lu and B. E. Douglas 1973 *Journal of the Acoustical Society of America* **54**, 74–79. Forced response of a discontinuously constrained damped ring.
14. W. S. Cramer 1957 *Journal of Polymer Science* **26**, 57–65. Propagation of stress waves in rubber rods.
15. G. C. Everstine and J. M. McKee 1974 *A Survey of Pre- and Post-Processors for NASTRAN* (Editors W. Pilkey, K. Saczalski and H. Schaeffer). University Press of Virginia, Charlottesville, Virginia. Structural mechanics computer programs: surveys, assessments and availability.
16. G. C. Everstine 1975 *NASA TM X*-3278, 511–521. Recent improvements to BANDIT, NASTRAN: users' experiences.