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MORE ON FINITE ELEMENT MODELING OF DAMPED COMPOSITE SYSTEMS

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Finite element procedures are developed and verified for layered beams and rings having either continuously or discontinuously constrained viscoelastic damping layers. The two configurations considered are (1) a three-layered sandwich beam or ring (closed curved beam) consisting of two thin elastic layers with a viscoelastic core in between, and (2) a damped composite made of a thin-walled elastic structure having a finite number of mass segments or elastic segments adhered to it by a viscoelastic material. Viscoelastic material dependence on frequency and temperature is accounted for. Numerical predictions of transverse driving point impedances agree very well with available experimental data.

1. INTRODUCTION

Structural design objectives such as low noise, low weight, long life, and high levels of reliability are becoming harder to achieve by using conventional structural design techniques because of ever-tightening requirements. However, it has long been observed that structural noise and vibration can be reduced by incorporating high energy dissipating mechanisms into the structural fabrication: i.e., by using layers of viscoelastic shear damping material to dissipate energy within vibrating members themselves. One approach is to induce a shear strain in the damping material by bonding it between two layers of much stiffer materials to form a sandwich [1–5] in which most of the shear deformation occurring during flexure of the sandwich will be in the middle layer consisting of the more compliant damping material.

Other damping arrangements in which the same principle is employed include damping tapes, spaced damped treatment, laminated structures, and strip dampers, all of which are based on the cell-insert design concept conceived by Ruzicka [6, 7]. Ungar and Kerwin [8] referred to such treatments as constrained viscoelastic layer systems. The purpose of all the various treatments is to provide effective damping over a wide range of frequencies for given engineering requirements and applications. The good structural, fatigue, and acoustic properties of these lightweight structures, as well as their remarkable ability to dampen vibrations, have resulted in their rapid development for use in aircraft and other industries.

The vibrations of continuously and discontinuously constrained damped structures have attracted the attention of investigators for many years. Analyses of simple structural elements such as beams [2–4, 9, 10], rings [11–13] and plates [9, 10, 14, 15] have indicated that the complexity of closed-form analyses will drastically increase, and that solutions for more complicated and practical systems of this nature will be difficult to obtain. It is thus natural that finite element approaches would appear. Although it seems clear that these layered members could be modeled as non-homogeneous *three*-dimensional continua, such an approach is not only expensive but also unnecessary. Instead, what are desired are schemes which are simple (and perhaps even approximate) but which adequately account

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for stiffness, inertia and damping effects. For example, Mau and Tong [16] recently demonstrated the feasibility and accuracy of a hybrid finite element solution for a continuously constrained damped sandwich ring. Also, the present authors have obtained finite element solutions for vibrations of three-layered damped sandwich plate composites [17] using an approach in which the damping core is modeled by a single layer of linear isoparametric solid finite elements and the constraining layers by rigidly attached plates.

The purpose of this paper is to present practical finite element approaches for two additional structural configurations: beams and rings having the damping layer constrained either continuously or discontinuously. The various types of damped structural configurations are shown in Figure 1. The finite element approach used for the continuously constrained beam (Figure 1(b)) and ring (Figure 1(c)) is analogous to that presented previously [17] for the damped plate (Figure 1(a)). The aim is thus to establish that an analogous procedure works for these configurations as well.

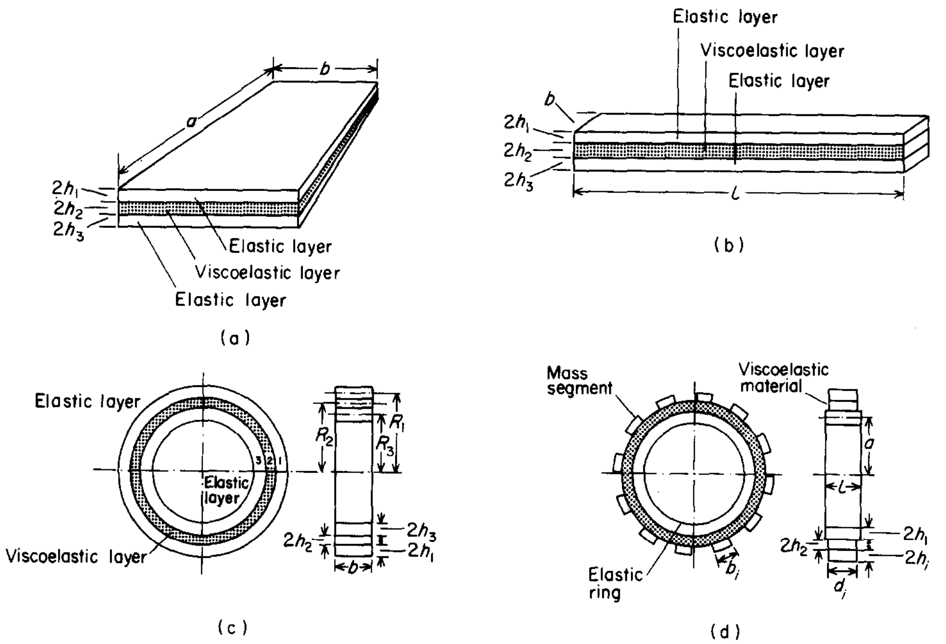

Figure 1. Damped composite structures.

For the discontinuously constrained damped ring (Figure 1(d)), the finite element approach used here for the viscoelastic layer and attached mass segments is based on a closed-form approach developed previously [12].

2. CONTINUOUSLY CONSTRAINED DAMPED BEAM AND RING

The two continuously constrained system configurations considered are shown in Figures 1(b) and 1(c). Various geometrical notations are indicated in these figures.

Each system considered has free boundaries and is subjected to a concentrated load. For the beam, the load is applied transversely at the midsection of the beam. For the ring, the load is applied radially at a point on the surface. Thus all cases possess symmetry.

In an earlier analysis [17] of a damped plate (Figure 1(a)) the viscoelastic core was modeled by a single layer of linear isoparametric solid elements, having plates rigidly attached to each face, to model the constraining layers. This approach can be simplified for application to the one-dimensional damped beam structure of interest here. For these structures the viscoelastic damping core is modeled by a single layer of linear isoparametric quadrilateral membrane elements which are the counterparts of the isoparametric quadrilateral brick elements used for the plate composites. The upper and lower elastic beam layers are modeled by beam elements which are eccentric with respect to the defining grid points. Therefore, there are only two layers of grid points, one on the upper and one on the lower surface of the viscoelastic layer. The beam specimen incorporated a 0·004 inch (0·01 cm) acrylic base viscoelastic polymer material, with mass density 0·0398 lbm/in^3 (1·102 g/cm^3), sandwiched between two identical machined steel beams $\frac{1}{4}$ inch (0·635 cm) thick, 1 inch (2·54 cm) wide, and $24\frac{3}{16}$ inches (61·436 cm) long. The complex shear modulus G of the viscoelastic layer for the frequency range of 50 to 1000 Hz at 124°F (51·1°C) can be approximated by

$$G = G_0(1 + i\beta), \tag{1}$$

where

$$G_0 = 3{\cdot}114 f^{0{\cdot}579}\text{psi}(=21{\cdot}47 f^{0{\cdot}579}\,\text{kPa}),\ \beta = 1{\cdot}008 f^{0{\cdot}022}, \tag{2, 3}$$

with the frequency f in Hz. In modeling the structure, 12 isoparametric quadrilateral membrane elements and twelve eccentric beam elements are used for half the length of the composite.

The viscoelastic material used here is assumed to be nearly incompressible, which is approximately true for most viscoelastic materials [18]. It is also assumed that the loss factors in shear and in direct deformation are equal. Poisson's ratio is taken as 0·48 to avoid numerical difficulties. The steel layers are assumed to be lossless.

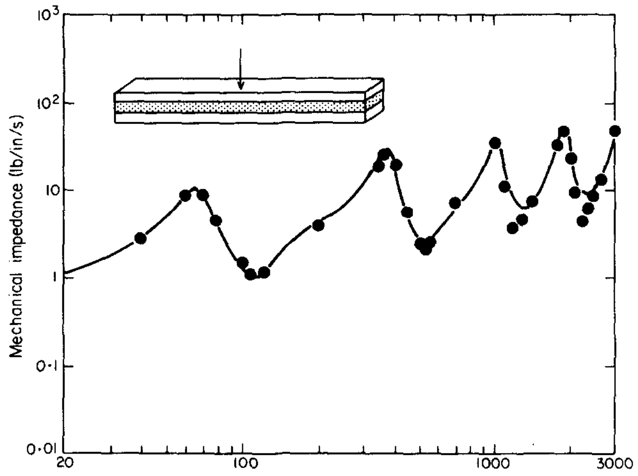

Figure 2. Mechanical driving point impedance of a damped beam. ———, Experimental data; ●●●, NASTRAN solutions; 1 lb/in/s = 175·13 N/m/s.

Presented in Figure 2 is a comparison of the experimental results [19] and the finite element solutions obtained for this three-layer sandwich beam. The numerical results were obtained by using the NASTRAN [20] program for time-harmonic forced response. The agreement between the two sets of results in Figure 2 is clearly very good.

The second structural configuration considered was the continuously constrained damped ring. The finite element modeling approach here was the same as for the straight beam. One layer of 12 linear isoparametric membrane elements was sandwiched between two layers of 12 eccentric beam elements. Due to symmetry, only 180° of the ring was

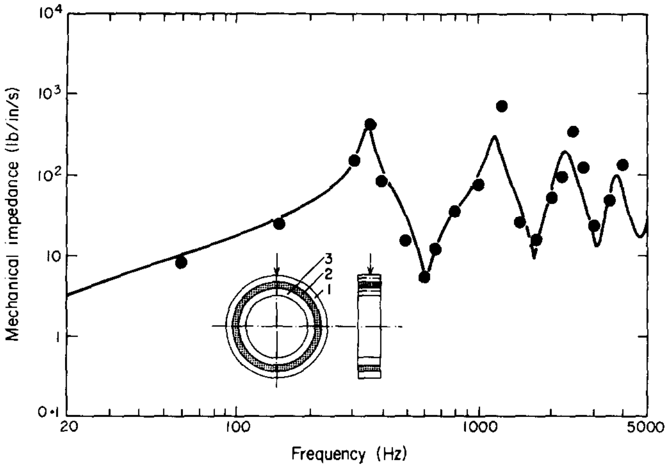

Figure 3. Mechanical driving point impedances of a damped ring. ——, Experimental data; ● ● ●, NASTRAN solutions; 1 lb/in/s = 175·13 N/m/s.

modeled. Only two rows of grid points are required, one on the outer and one on the inner surface of the viscoelastic layer. In Figure 3, the mechanical driving point impedances of a three-layered continuously constrained damped ring are presented. The specimen incorporated a viscoelastic core with mass density 0·0398 lbm/in^3 (1·102 g/cm^3) sandwiched between two steel rings. The ambient temperature is 72°F (22·2°C), and the complex shear modulus of the viscoelastic material at this temperature in the frequency range of 40 to 4000 Hz can be approximated by

$$G_0 = 20{\cdot}53 f^{0{\cdot}494}\ \mathrm{psi}\ (=141{\cdot}6 f^{0{\cdot}494}\ \mathrm{kPa}),\quad \beta = 1{\cdot}46. \qquad (4, 5)$$

The geometrical dimensions of the system are as follows: R_1 = 4·3538 inches (11·06 cm), $2h_1$ = 0·0747 inch (0·190 cm), $2h_2$ = 0·004 inch (0·010 cm), $2h_3$ = 0·25 inch (0·635 cm), b = 2 inches (5·04 cm). The agreement between the present numerical results and the test data [13] is clearly very good.

3. DISCONTINUOUSLY CONSTRAINED DAMPED RING

The discontinuously constrained system configuration considered is a thin-walled ring such as shown in Figure 1(d), having a finite number of mass segments attached uniformly to its circumference by a viscoelastic layer. This particular ring has previously been used for some analytic closed-form solutions [11, 12]. In this damped system, the three parts considered are the elastic ring, the attached mass segments, and the viscoelastic layer connecting the ring and the attached mass segments. The viscoelastic material is assumed to act as a complex spring between the ring and the adjacent attached mass segments. Each attached mass segment is considered to act as a single degree of freedom system. The equations of motion in the radial and circumferential directions can be written as

$$-m_i\Omega^2 w_i + k^*[w_i - w_t(\theta_i)] = 0, \quad -m_i\Omega^2 v_i + k_s^*[v_i - v_t(\theta_i)] = 0, \qquad (6, 7)$$

where $\Omega = 2\pi f$ is the radian frequency of excitation, and w_i and v_i are the radial and circumferential displacement components, respectively, of mass segment i of mass m_i. The quantities $w_t(\theta_i)$ and $v_t(\theta_i)$, which are unknown and yet to be determined, are the radial and circumferential components of displacement at the point of attachment of mass segment i. The equivalent extensional and shear spring constants k^* and k_s^* depend on the dimensions of the mass segment, the thickness of the viscoelastic layer, and the material properties of the viscoelastic material. The viscoelastic material properties, which are complex, are both temperature- and frequency-dependent. The spring constants are

$$k^* = E^* b_i d_i/(2h_2), \quad k_s^* = G^* b_i d_i/(2h_2), \tag{8, 9}$$

where b_i is the circumferential length of mass segment i, and d_i is the width. The viscoelastic layer has a complex Young's modulus of elasticity E^*, a complex shear modulus G^*, and a thickness $2h_2$. The displacements of such a damped system can be determined by solving equations (6)–(9), along with the standard finite element matrix equation for the ring, the appropriate boundary conditions, and the loading.

The ring composite analyzed consisted of a steel elastic ring having 12 identical mass segments adhered to it by a viscoelastic material. The mass segments are also made of steel and are assumed to be uniformly distributed around the circumference of the ring. The viscoelastic layer is an acrylic base material with a complex shear modulus at 75°F (25°C) that can be approximated for the frequency range of interest by

$$G_0 = 20{\cdot}53 f^{0{\cdot}5}\ \text{psi}\ (=141{\cdot}6 f^{0{\cdot}5}\ \text{kPa}), \quad \beta = 1{\cdot}46. \tag{10, 11}$$

The geometrical dimensions of the system (see Figure 1(d)) are as follows: $l = 3$ inches (7·62 cm), $2h_1 = 0{\cdot}5$ inch (1·27 cm), $a = 4{\cdot}0625$ inches (10·32 cm), $2h_2 = 0{\cdot}004$ inch ($1{\cdot}02 \times 10^{-2}$ cm), $b_i = 2$ inches (5·08 cm), $d_i = 3$ inches (7·62 cm), $2h_i = 0{\cdot}5$ inch (1·27 cm), $\theta_i = (2i - 1)\pi/N$, $i = 1, 2, \ldots, N$, and $N = 12$.

In modeling the structure, general quadrilateral plate elements are used for the elastic ring. For generality, the attached mass segments are modeled by beam elements instead of as non-structural mass. The thin viscoelastic layer is approximated by viscoelastic beams connecting the mass segments and the ring. The structural damping for these beams can be specified to simulate the shear and extensional damping provided by the thin viscoelastic layer. By symmetry, only one half of the composite ring circumference is modeled. Two rows of 12 identical plate elements are used in the circumferential direction for the elastic ring; two rows of six identical longitudinal beam elements model the attached masses; and

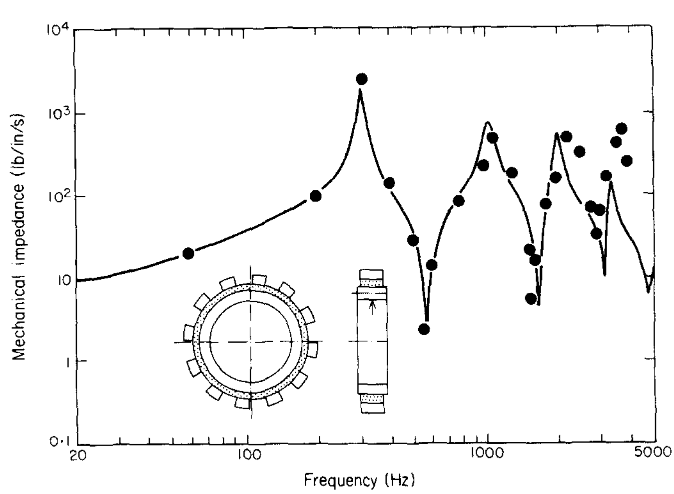

Figure 4. Mechanical driving point impedances of a discontinuously constrained damped ring. ———, Experimental data; ● ● ●, NASTRAN solutions; 1 lb/in/s = 175·13 N/m/s.

three radially oriented beam elements connect each of the attached mass segments to the ring for the thin viscoelastic layer. The load is applied at the center node of the first plate element on the top of the model. Presented in Figure 4 are the mechanical driving point impedances of this system. As with the previous comparisons presented, the numerical results agree very well with the experimental data. Certainly, the finite element modeling for the continuously constrained damped ring given in the previous section can be applied to this configuration.

4. DISCUSSION AND CONCLUSIONS

The numerical solutions given in Figures 2 through 4 indicate that the results obtained for these damped systems compare well with the available experimental data. It is likely that the finite element solutions could be further improved by using finer meshes. Although we are dealing with simple geometries in this paper, the significance of these results is that they demonstrate that general finite element programs (such as NASTRAN [20]) can be used, as is, to solve complex and practical problems of damped structures.

The major difference between the finite element method analysis of these damped systems and that of similar elastic *undamped* layered structures is that the frequency-dependence of the viscoelastic cores precludes the use of modal solutions of the frequency response problem. The modal approach is generally preferred to a direct solution of the matrix equations whenever a large number of excitation frequencies is involved.

The finite element formulation based on the damped plate model includes the effects of transverse shear flexibility, translatory and transverse inertias (both in the face plates and the core), and extensional effects in the viscoelastic core. It is not assumed, however, that the normal displacements of the elastic and viscoelastic layers are equal. Other assumptions common to closed-form analyses, such as equal facing thicknesses, small damping, and small bending stress in the core, are also removed.

Although only thin viscoelastic cores are considered here, there is no particular limitation on core thickness implicit in the approach, because the continuously constrained cores have been modeled by linear isoparametric elements also suitable for thicker applications. In fact, the reason that the analysis succeeds with only a *single* layer of elements in the core is probably that the only role expected of the core is to shear properly (so that the damping is correct) and to hold the constraining layers together.

Finally, the formulation can be applied to more general damped structures such as curved plate configurations, or to structures with partial covering damping treatment. The approach used for the discontinuously constrained damped ring can also be easily extended to a damped cylindrical shell system in which the mass segments are discretely distributed around the outer circumference at an arbitrary section of the shell.

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